

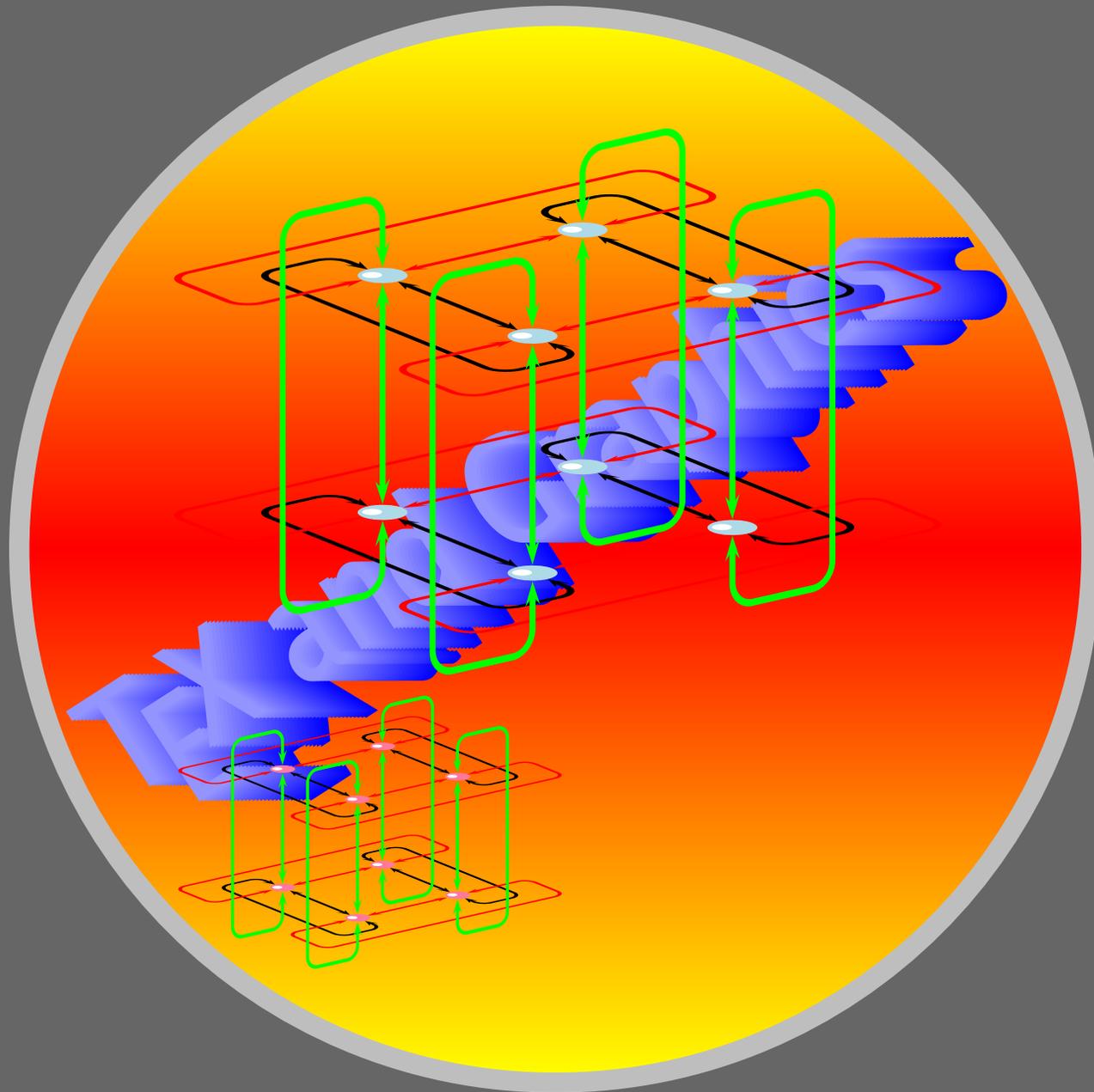
# Curvy Tricks

*Open and closed curves*

*Invisible ends*

*Curve tweaking*

*A new curve*



## Online $\text{\LaTeX}$ Tutorial Part II - Graphics PSTricks

E Krishnan, CV Radhakrishnan and AJ Alex constitute the graphics tutorial team. Comments and suggestions may be mailed to [tutorialteam@tug.org.in](mailto:tutorialteam@tug.org.in)

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## 4. Curvy Tricks

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We've seen how the `\psline` command can be used to form paths and how the `\pspolygon` path can be used to form closed paths joining specified points. Both these commands produce paths made up of pieces of *straight lines*. We can also produce *curves* joining specified points. This chapter explains how.

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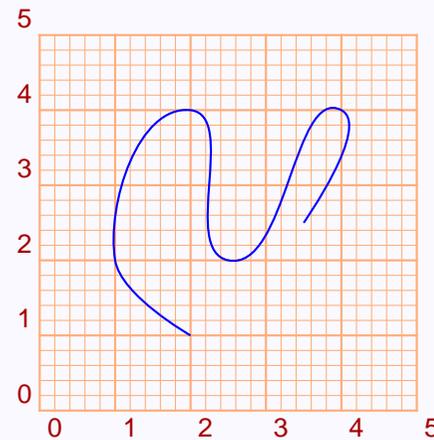
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## 4.1. Open and closed curves

To produce an “open” curve joining specified points (analogous to `\psline`), we use the `\pscurve` command. Look at the example below:

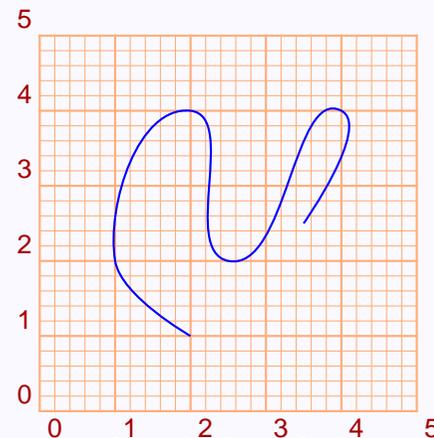
```
\begin{pspicture}(0,0)(5,5)
  \pscurve[linecolor=Blue]%
    (2,1)(1,2)(2,4)%
    (2.5,2)(4,4)(3.5,2.5)
\end{pspicture}
```



As in the earlier examples, the grid is *not* drawn by the code shown here.

The points specified can be shown in the picture by setting the parameter `showpoints` to true as shown below:

```
\begin{pspicture}(0,0)(5,5)
  \pscurve[linecolor=Blue,%
    showpoints=true]%
    (2,1)(1,2)(2,4)%
    (2.5,2)(4,4)(3.5,2.5)
\end{pspicture}
```



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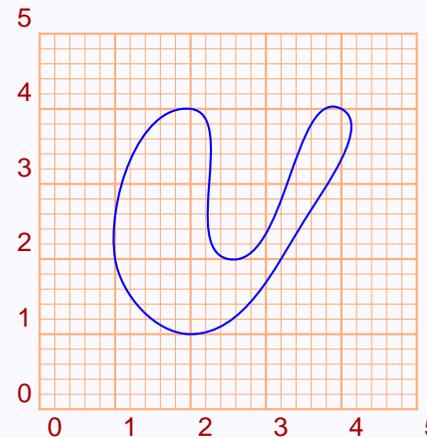


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A “closed” curve joining specified points is produced by the `\psccurve`. (Note the extra `c` in the middle. It stands for “closed”). The same points in the above example are used to form a closed curve in the next example:

```
\begin{pspicture}(0,0)(5,5)
  \psccurve[linecolor=Blue]%
    (2,1)(1,2)(2,4)%
    (2.5,2)(4,4)(3.5,2.5)
\end{pspicture}
```



As we know, we can draw infinitely many curves through a set of specified points. So, what’s the peculiarity of the curve that `\pscurve` (or `\psccurve`) produces? Well, it’s like this: if  $A$ ,  $B$ ,  $C$  are three *consecutive* points of the specified set, then the curve is drawn such that at  $B$  (the middle point), the curve (or more precisely, the tangent to the curve) is perpendicular to the bisector of angle  $ABC$ . Perhaps this is better described by a picture. The picture is a magnified version of the open curve we’ve drawn above with the first three points marked as  $A$ ,  $B$ ,  $C$  and showing the bisector of angle  $ABC$  and its perpendicular at  $B$  (which you can see is the tangent to the curve at  $C$ ).

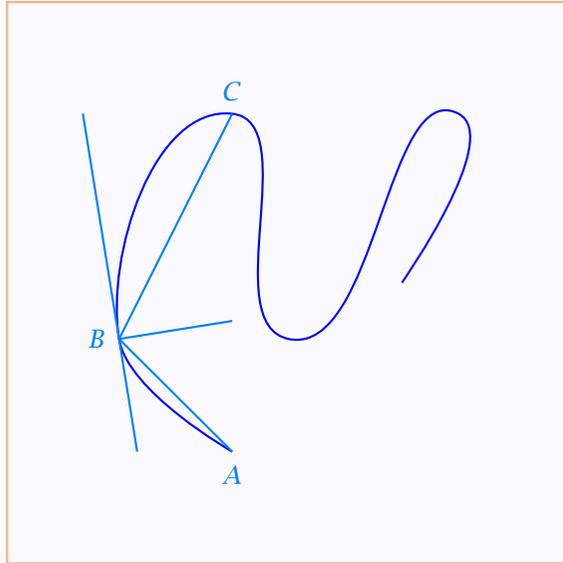
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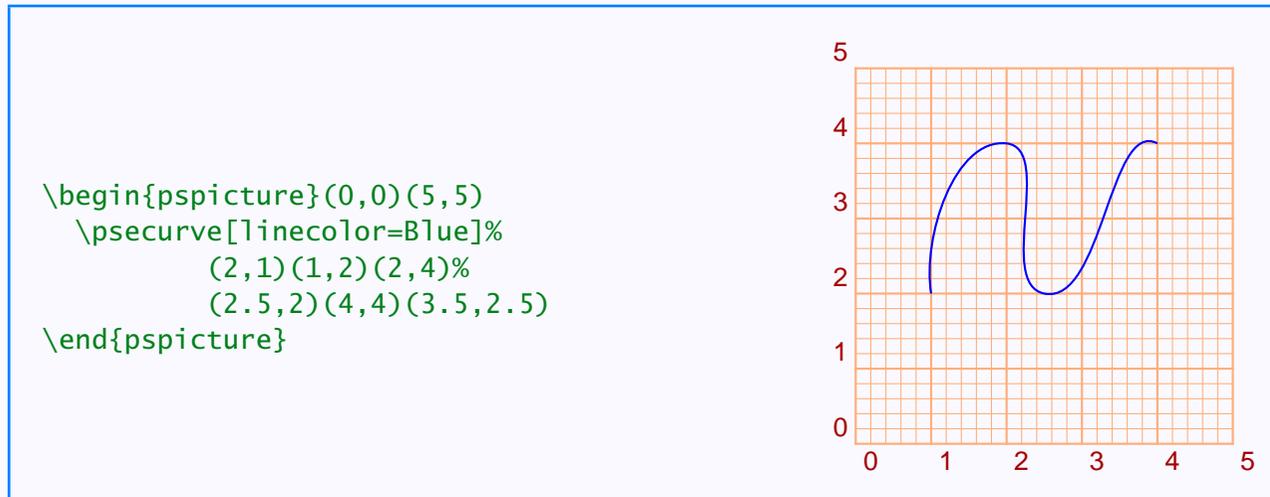


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## 4.2. Invisible ends

there's a third command to form a curve, namely, `\psecurve`, which draws a curve through the specified points *excluding the first and the last* (The `e` in the middle suggests “endpoints” . Thus the same set of points we used above joined using this command gives the picture below:



Now what's the use of specifying some points, if you are not using them in the picture? The fact is, though `\psecurve` does not *draw* the curve through the endpoints, it does *see* them. The invisible end points determine the *shape* of the curve at the visible extremities. Look at the example below:

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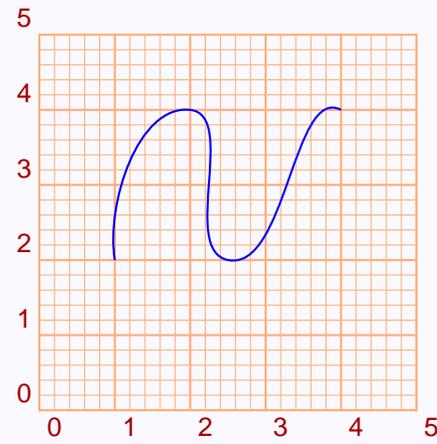
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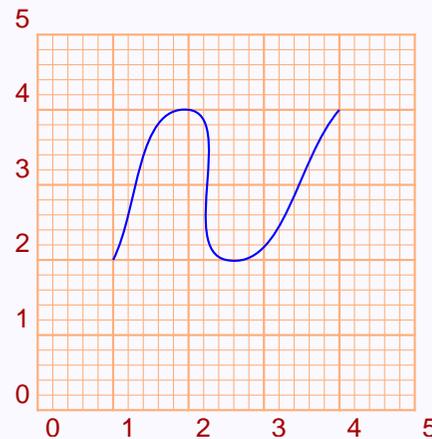
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```
\begin{pspicture}(0,0)(5,5)
  \psecurve[linecolor=Blue]%
    (2,1)(1,2)(2,4)%
    (2.5,2)(4,4)(3.5,2.5)
\end{pspicture}
```



```
\begin{pspicture}(0,0)(5,5)
  \psecurve[linecolor=Blue]%
    (0.5,1)(1,2)(2,4)%
    (2.5,2)(4,4)(4.5,4.5)
\end{pspicture}
```



Perhaps the influence of the invisible endpoints is better illustrated, if we draw both the  $\text{\pscurve}$  and  $\text{\psecurve}$  together. The next picture does this for the two sets of points in the above example, where the portion of the  $\text{\pscurve}$  not drawn by  $\text{\psecurve}$  is shown in a lighter color.

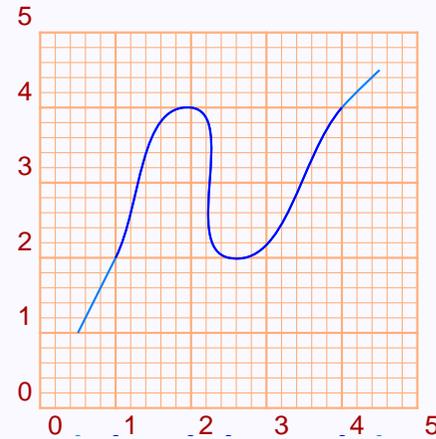
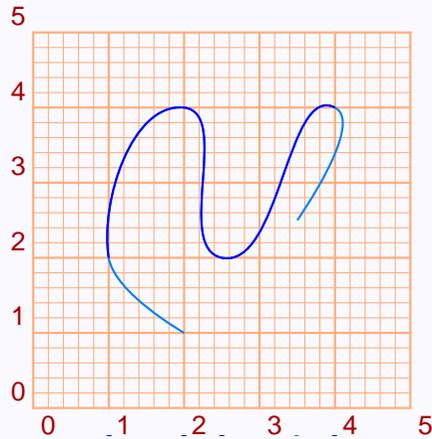
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### 4.3. Curve tweaking

The `\psecurve` is one way of changing the default shape of the curve that `\pscurve` produces. There are also ways to change the “tautness” of the curve and also its “slope”. This is done by changing the values of the curvature parameter. Its value is a triplet of numbers, with the “useful” values of each ranging from  $-1$  to  $2$ . We specify the values by setting

$$\text{curvature}=\textit{number1 number2 number3}$$

The default set of values for these three numbers are  $1$ ,  $0.1$  and  $0$ . We’ll look at each of these values separately to see what each signifies.

The first number of curvature determines the tautness of the curve. As mentioned above, its default value is  $1$ . Lower values make the curve tighter and higher values makes it looser. The pictures below, with the values of the curvature given underneath, illustrates this. The first picture for example is produced by

```
\pscurve[linecolor=Blue,%
  showpoints=true,%
  curvature=2 0.1 0]%
(2,1)(1,2)(2,4)%
(2.5,2)(4,4)(3.5,2.5)
```

Note that *only* the first number in the curvature is changed in these four curves, the others retaining their default values. The value  $0$  for this number makes `\pscurve` join every pair of points with straight lines (so that it becomes `\psline`) and further tightening through higher values bents it out of shape.

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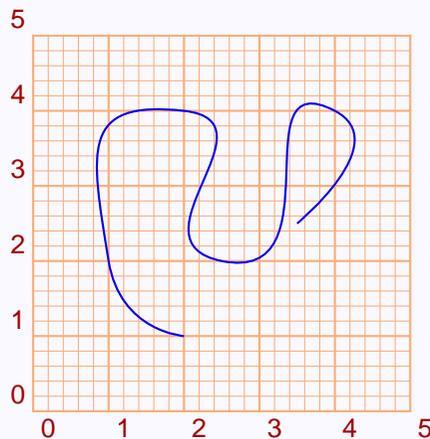
Open and closed curves

Invisible ends

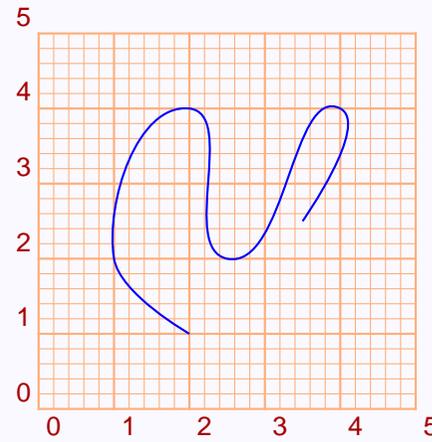
Curve tweaking

A new curve

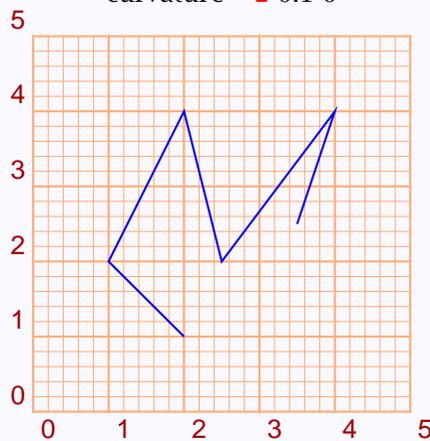
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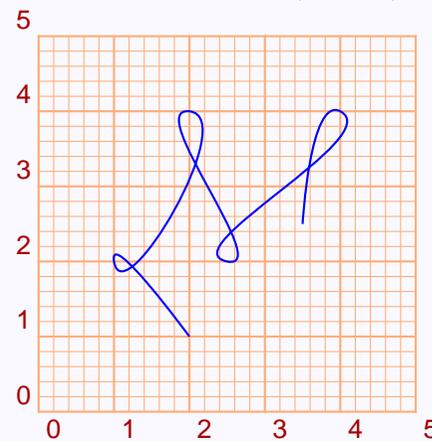
curvature = 2 0.1 0



curvature = 1 0.1 0 (Default)



curvature = 0 0.1 0



curvature = -1 0.1 0

The second number of curvature is again for tightening or loosening the curve, but only the portions between certain pairs of points. Suppose  $A, B, C$  are three consecutive points specified. If angle  $ABC$  is less than 45 degrees, then lower values for this second number makes the curve tighter *around B*, and higher values makes it looser around  $B$ . If the angle is greater than 45 degrees, it does the reverse. The next four pictures illustrate this behavior for the set of points we have been using in all our examples. Note that for these points the angle described above is greater than 45 degrees

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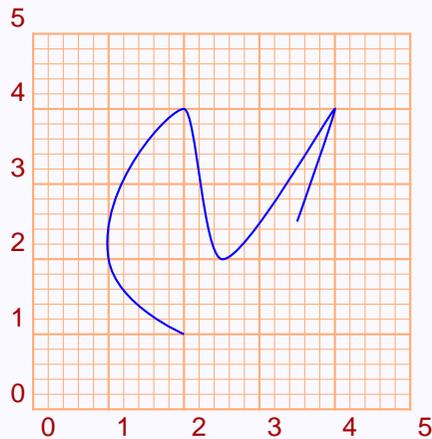
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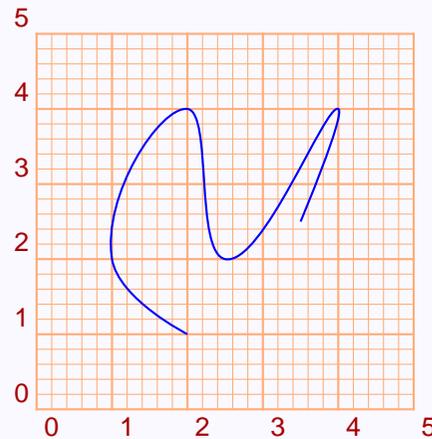
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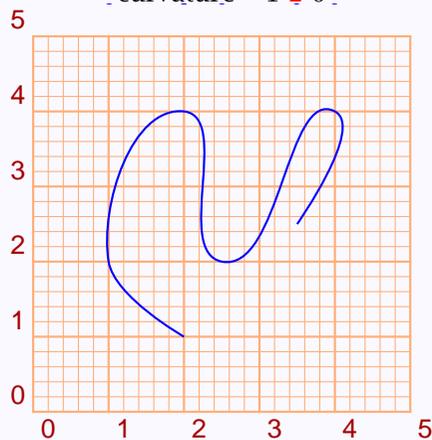
only at the second point.



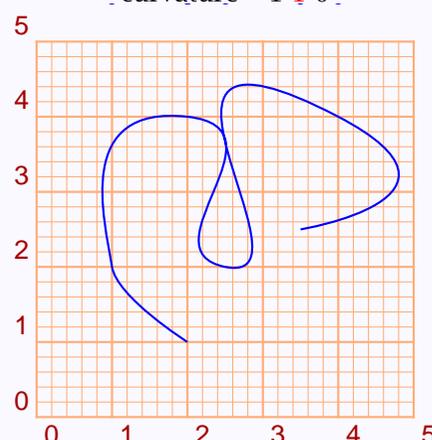
`curvature = 1 2 0`



`curvature = 1 1 0`



`curvature = 1 0.1 0 (Default)`



`curvature = 1 -1 0`

Note how the curve becomes increasingly taut at the second point and increasingly slack elsewhere as the second number decreases.

The third value of `curvature` controls the “slope” of the curve at each point—that is, the angle the tangent at each point makes with the horizontal. We had mentioned earlier that for the curve produced by `\pscurve`, if  $A, B, C$  are three consecutive points, then the tangent at  $B$  is perpendicular to the bisector of angle  $ABC$ . Actually, this is true only for the default

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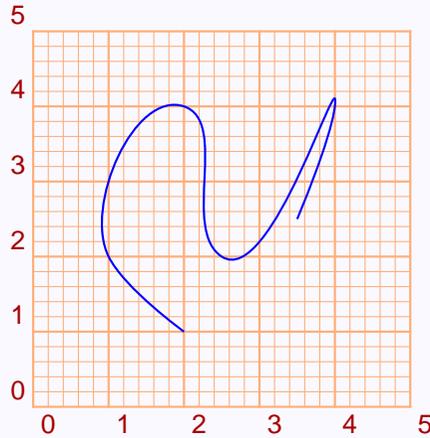
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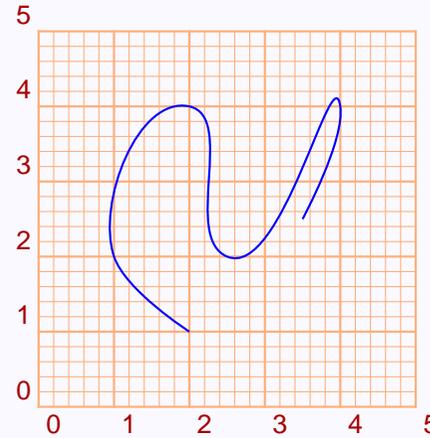
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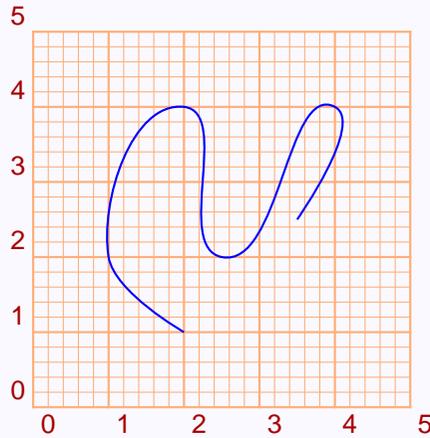
value of 0 for the third number in curvature. The value  $-1$ , for example, makes the tangent at  $B$  parallel to the line  $AC$ . The pictures below show the changes in the curve produced by varying this number.



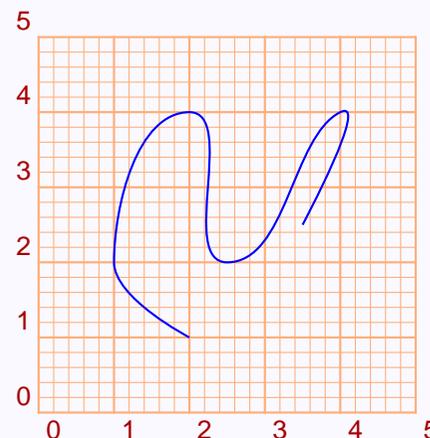
curvature = 1 0.1 2



curvature = 1 0.1 1



curvature = 1 0.1 0 (Default)



curvature = 1 0.1 -1

Perhaps, the effect will be better seen, if we draw the tangents also at some point. We do this in the next four pictures, taking the last three points as a sample:

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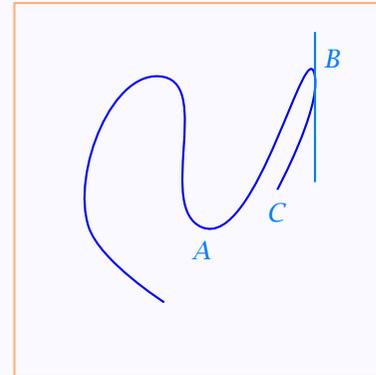
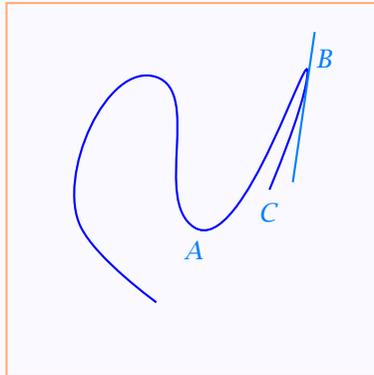
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Open and closed curves

Invisible ends

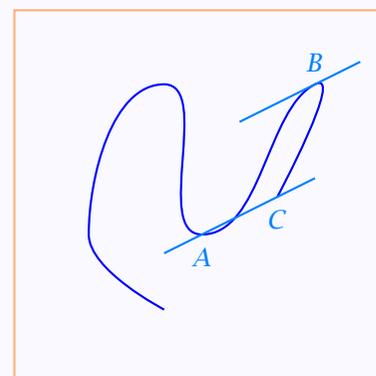
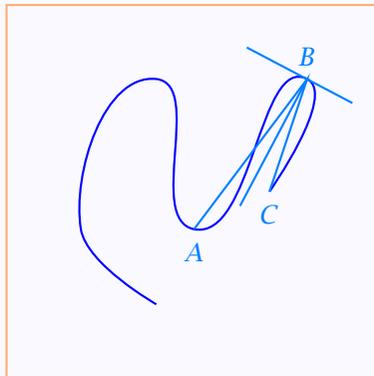
Curve tweaking

A new curve



curvature = 1 0.1 2

curvature = 1 0.1 1



curvature = 1 0.1 0 (Default)

curvature = 1 0.1 -1

Note how decreasing the third number rotates the tangent at  $B$  counter-clockwise.

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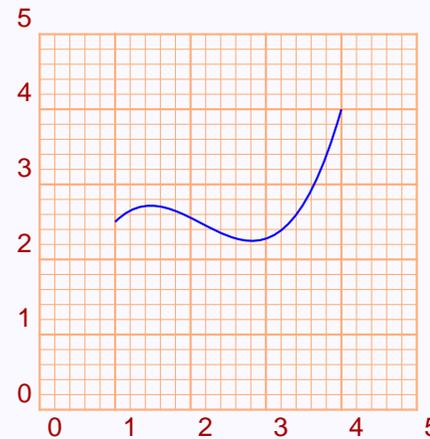
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## 4.4. A new curve

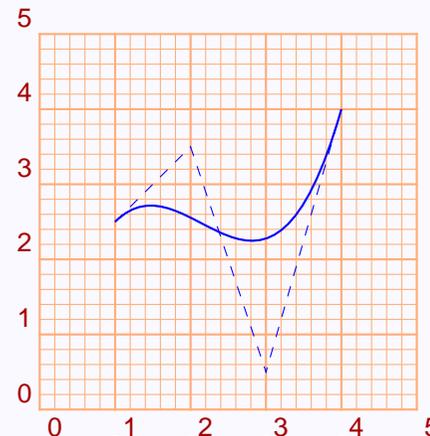
Another type of curve we can draw with PSTricks is the so called Bézier curve (named after Pierre Bézier, who realized their importance in computer-aided design). This is done by the command `\psbezier` command and we must specify four “control” points. Look at the example below:

```
\begin{pspicture}(0,-1)(5,6)
  \psbezier[linecolor=Blue]%
    (1,2.5)(2,3.5)%
    (3,0.5)(4,4)
\end{pspicture}
```



Let's try `showpoints=true` on this.

```
\begin{pspicture}(0,-1)(5,6)
  \psbezier[linecolor=Blue,%
    showpoints=true]%
    (1,2.5)(2,3.5)%
    (3,0.5)(4,4)
\end{pspicture}
```



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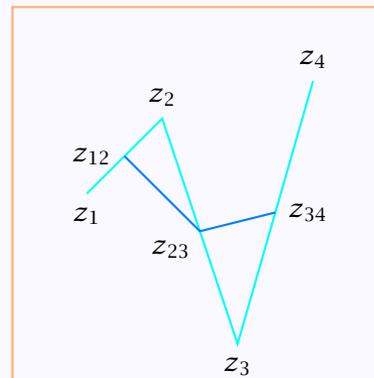
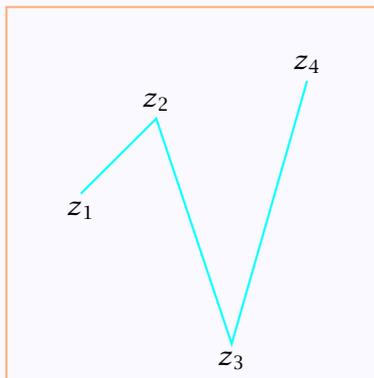


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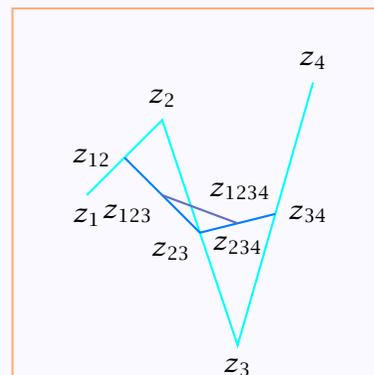
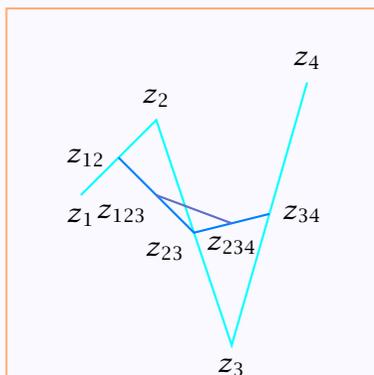
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You can see that the curve starts at the first point specified, tangent to the line joining this point to the second and ends at the last point, tangent to the the line joining this point and the third. The second and third points, in addition to determining the tangency at the endpoints, also exert a “pull” on the curve. Let’s see in detail how the curve is actually drawn.

Suppose the control points are  $z_1, z_2, z_3, z_4$ . First they are joined by straight lines and the midpoints of each segment is found. Let’s call them  $z_{12}, z_{23}$  and  $z_{34}$



These three points are again joined by straight lines and then the midpoints of the two line segments are found. Let them be  $z_{123}$  and  $z_{234}$ . These two points are joined by a straight line and the mid point of this line segment is found.



## Online $\text{\LaTeX}$ Tutorial Part II - Graphics PSTricks

E Krishnan, CV Radhakrishnan and AJ Alex constitute the graphics tutorial team. Comments and suggestions may be mailed to [tutorialteam@tug.org.in](mailto:tutorialteam@tug.org.in)

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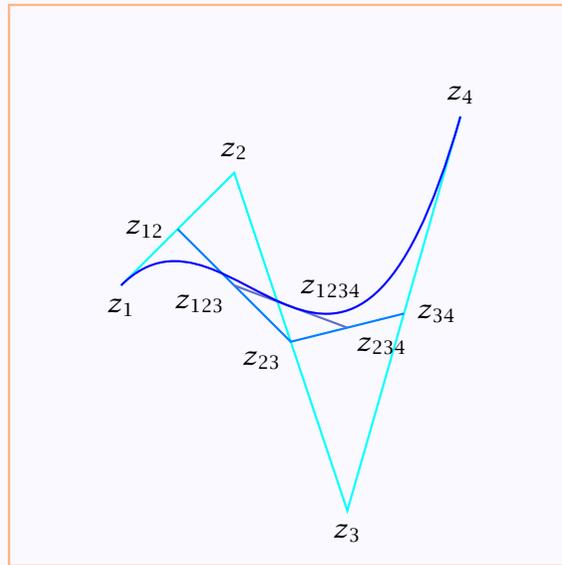


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This point, which we denote by  $z_{1234}$  is a point on the curve as can be seen in the next (enlarged) picture.



How about the other points of the curve? Well, the process we've described above is repeated with the new control points  $z_1, z_{12}, z_{123}, z_{1234}$  and  $z_{1234}, z_{234}, z_{34}, z_4$  to get another point and so on *ad infinitum*. This process converges rather quickly and the scaffolding is finally removed.<sup>1</sup>

<sup>1</sup>For the mathematically inclined, it may be interesting to note that the curve can be algebraically described by the equation

$$z(t) = (1-t)^3 z_1 + 3(1-t)^2 t z_2 + 3(1-t)t^2 z_3 + t^3 z_4$$

a *Bernshtein polynomial* of degree 3.

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